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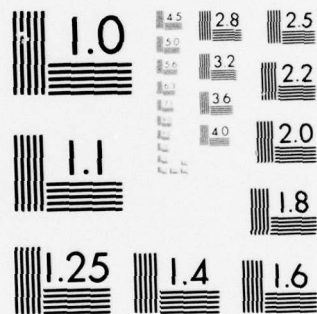
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How Representative Is An Oceanic Temperature Observation ?

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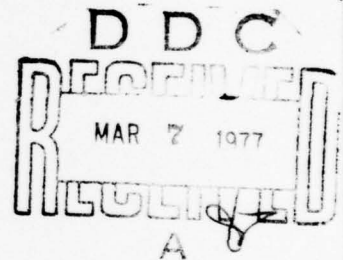
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TEMPERATURE OBSERVATION?

42 28p.

15 Nonr-3348(00)



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FOREWORD

This research was carried out under the independent research program of the Oceanography Department, Lockheed-California Company. The gathering of the basic data used in this report was supported in part by the Office of Naval Research under Contract NOnr 3348(00). Permission to use these data is greatly appreciated.

The authors especially thank Mr. J. F. T. Saur, of the Bureau of Commerical Fisheries, for critical review and many helpful suggestions.

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TABLE OF CONTENTS

	PAGE
FOREWORD	ii
TABLE OF CONTENTS	iii
LIST OF FIGURES	iv
ABSTRACT	v
I - INTRODUCTION	1
II - COMPUTATIONS AND DISCUSSIONS	4
OBSERVATIONAL DATA	4
VARIABILITY	4
DISTRIBUTION	9
COHERENCE	12
ALIASING	13
RELIABILITY OF ESTIMATES	14
III - CONCLUSIONS	18
REFERENCES	21

LIST OF FIGURES

FIGURE		PAGE
1	Sites and Exposures of the Observations. The dotted area delineates depths greater than 1000 m.	5
2	Typical Temperature Change as a Function of Time Lag Between Observations. Case 1. $S(L)$ is the root-mean-square successive deviation, $\langle \Delta T(L) \rangle$ is the mean absolute successive difference.	7
3	Typical Temperature Change as a Function of Time Lag Between Observations. Case 2. See Fig. 2 for explanation.	7
4	Autocorrelation of the Temperature-Time Series. Case 1	8
5	Autocorrelation of the Temperature-Time Series. Case 2	8
6	Typical Hourly Mean Temperature Profiles. Case 1. The small numbers show the standard deviation of the observations over the hour. Dashed lines are used to connect regions of missing data.	10
7	Typical Hourly Mean Temperature Profiles. Case 2. See Fig. 6 for explanation.	10
8	Typical Frequency Distributions of Temperature Changes for Time Lags of 10, 90, and 300 Minutes. Case 1.	11
9	Typical Frequency Distribution of Temperature Changes for Time Lags of 10, 90, and 300 Minutes. Case 2.	11
10	Autocorrelation of the Filtered Temperature-Time Series. Case 2.	15
11	Comparison of Filtered and Unfiltered Root-Mean-Square Successive Deviations. Case 2. $S(L)$ and $S(K)$ are the root-mean-square successive differences based on the instantaneous readings and hourly averages respectively. $\langle \Delta T(L) \rangle$ and $\langle \Delta T(K) \rangle$ are the equivalent respective mean absolute deviations.	15
12	Temperature Change as a Function of Time Lag Between Observations That Was Not Exceeded in 80% of the Observations. Case 1.	19
13	Temperature Change as a Function of Time Lag Between Observations That Was Not Exceeded in 80% of the Observations. Case 2.	19

ABSTRACT

The representativeness of a temperature observation is studied by measuring the deterioration of the reliability of the observation with time. The rate of deterioration or amount of temperature change as a function of time is described statistically for two sets of data in terms of the root-mean-square successive deviation, the mean absolute deviation, the autocorrelation, and the frequency distribution of the deviations. The small-scale deviations are shown to be large enough to require their consideration in the design of synoptic measurement systems and many other applications. In one extreme case 20% of the temperature changes over 5 hr were greater than about

$\pm 3^{\circ}\text{C}.$

+ or -

SECTION I

INTRODUCTION

A single oceanographic measurement taken at one particular location and time does not precisely describe the state of the ocean at any other location or time. This measurement provides only an estimate of the state of the ocean for a locale of unknown size and for an unknown duration of time, reliability of the estimate is dependent upon the degree of accuracy required. In recent years it has become increasingly important to learn the size area and duration of time for which a single measurement can be considered representative [Stommel, 1963; Baer and Hamm, 1963; Fofonoff, 1964; Webster, 1964; Petersen and Middleton, 1963]. The 10th Eastern Pacific Oceanic Conference meeting in October 1963 has also recognized this importance by appointing an ad hoc study committee. The representativeness of an observation will vary not only with different regions of the oceans but with different seasons, further complicating the problem. This problem is basic to all phases of physical oceanography.

This variability is particularly critical in many practical problems. For example, in the design of buoy systems for synoptic measurements, the areal spacing of the buoys and the sampling interval determine the maximum useful accuracy of the sensor elements. Thus, this question must be answered before the design specifications can be drawn for such a system. Inadequate specification becomes very costly in such a system. In the case of overspecification, the result is a high cost for the development of overdesigned sensors. In underspecification, the data collected is inadequate for the purpose intended.

Of the many important parameters, temperature is the easiest to measure. It seems reasonable, therefore, to study it first. Thus this paper is devoted to only one phase of the problem of temperature variability. To describe the representativeness of temperature observations, the high-frequency or small-scale variability must be considered as well as the longer-term variability. This is especially important since the small-scale features will alias into the longer wavelengths or lower frequencies and thus yield a misleading result. As will be described later, careful design of observation systems and analysis of data can alleviate this problem.

Many papers have been published on subsurface temperature variations in the oceans, for example: Seiwell, 1940; Defant, 1950; Leipper, 1955; Robinson, 1957; Roden, 1960; Arthur, 1960. However, there is little mention of that scale of variability which is much less than that of tidal period, yet greater than what might be called the turbulent scale. In this paper we show that large temperature fluctuations are found over short time intervals and explain these variations in terms of the interrelationships between time and accuracy. Data are not available to us for a study of the spatial scale.

To study the problem of "microstructure" along an acoustic path, Lieberman [1951] mounted a resistance thermometer and a thermocouple on a submarine. As the submarine moved with a speed of 1 to 2 m/sec at depths of 30 to 60 m, he recorded the ambient temperature. The system was said to resolve the temperatures at spatial separations as small as 10 cm. He showed the autocorrelation function of his data could be approximated by $R(L) = e^{-L/60}$ where L is the spatial lag in cm. He then defined 60 cm as the mean size of the inhomogeneities, and finally, developed a relationship between the autocorrelation function and the propagation of underwater sound. The thermal microstructure was shown to be an important factor. Lieberman did not consider the temporal variations but assumed that they were small compared with the spatial variations.

Other studies by Whitmarsh, Skudrzyk and Urick [1957] and Skudrzyk [1963] have also shown the importance and effect of patchiness. They report that observations taken off Key West, Florida, of the horizontal temperature variability fit Kolmogorov's equilibrium theory of turbulence. Variability on the order of $0.1^{\circ}\text{C}/60\text{ cm}$ has also been shown to exist in estuaries [Smithson, 1960]; and variability of several degrees over an hour or less has been observed in the Baltic [Kalle, 1942; Neumann, 1949].

LaFond and Moore [1960, 1961] described the results of bathythermograph observations taken at half-hourly intervals at several locations at different times of the year. They computed the mean deviations and autocorrelation function at several lags of the temperatures recorded at different depths. They concluded that temperature inhomogeneities or internal waves are significant over intervals of time as short as 30 min. They also showed important variations with depth and time of year. The autocorrelation function with respect to time off the California coast decreased with depth below the thermocline. This could have been caused either by a larger proportion of random motion being present or else the coherent motions being so small that the effect of random observational errors became relatively more pronounced. That is, the signal-to-noise ratio decreased with depth.

LaFond and Moore [1962] described the temperature variations recorded by a towed thermistor chain in order to study spatial temperature variations. Data were recorded continuously in analog form on a paper strip and read at small intervals corresponding to a distance of about 100 m or one-half minute; thus, resolution was of the order of 1 min or 200 m. Correlation functions and spectra were computed to show the combined effect of temporal and spatial variation since they were not separable.

Wolff [1963] has given information on surface temperature variability with the conclusion that the variability in both space and time is not large enough to be important for weather prediction. However, values of temperature fluctuations of the order of 0.5° C over 1 hr and 9 km are presented, which for the purposes of this paper, must be considered significant. Wolff sampled at 24-sec intervals, thus allowing relatively high frequencies to be studied, however he did not publish information on them.

Haurwitz, et al [1959] reported serial observations of temperature from several bottom-mounted resistance thermometers off Bermuda. The depths were 50 and 500 m which are, respectively, in the seasonal and permanent thermoclines. The interval of observation was variously 30 min and 5 min. They reported that since the high-frequency variations were so large, it is probable that the longer-period fluctuations of about a month recorded simultaneously by hydrographic casts at the deeper depths "are simply the result of sampling errors."

Carsola, et al [1963] recorded temperatures digitally at 1-min intervals from a chain of 12 thermistors which were equally spaced 15 to 66 m below the surface. To minimize the motions of the chain, a weight and damper were on one end of the cable and a series of small floats attached to the cable near the surface instead of the usual single, larger buoy. Thus, when a wave passed, one or more of the buoys which had been floating on the surface would be carried under. The net effect was to maintain near neutral buoyancy at all times without large vertical motions of the array. The recording cable was supported by neoprene floats to its terminus on Lockheed's R/V Sea Quest which was anchored in water as deep as 1300 m. The data from two of these five sets of observations are the basis for the present study. The spectra computed from these records had no regions close to zero. However, the amplitudes generally decreased toward higher frequencies. There was some evidence of a bump near 20 min. There was also a suggestion of seasonal variation since the spectra taken in different seasons were not the same.

SECTION II

COMPUTATIONS AND DISCUSSIONS

OBSERVATIONAL DATA

The statistical properties of the successive time variations of temperature at each depth could have been computed from the spectra published by Carsola, et al [1963]. However, in order not to be bound by the limited range of periods in the published spectra and to avoid complicated interpretations, original observations were used directly. Two cases were chosen: (1) that of August 21-23, 1962, in 1300 m in the Santa Catalina Basin, and (2) that of September 17-21, 1962, in 1200 m in the San Diego Trough, hereafter referred to as Cases 1 and 2, respectively. The locations of the two sets of observations are shown in Figure 1 along with the important geographical features of the area. The records were 3275 and 5554 min long, respectively. Carsola, et al [1963] used only 4604 min because of a gap in the record for Case 2, but the computations of this report are adjusted for the gap so as to utilize the entire record. The Väisälä period, which is a simple estimate of the natural frequency of stability oscillations, was computed as 1 to 5 min, with the lower periods nearer the surface.

VARIABILITY

Three basic statistical parameters were computed to show the variability of the phenomena. First, the mean absolute successive differences, $\langle \Delta T(L) \rangle$, and the root-mean-square successive differences, $S(L)$, were computed to find the expected or average temperature change over a given interval of time. These are given by the expressions:

$$\langle \Delta T(L) \rangle = \frac{1}{N(L)} \sum_{i=1}^{N(L)} |T(i) - T(i+L)|$$

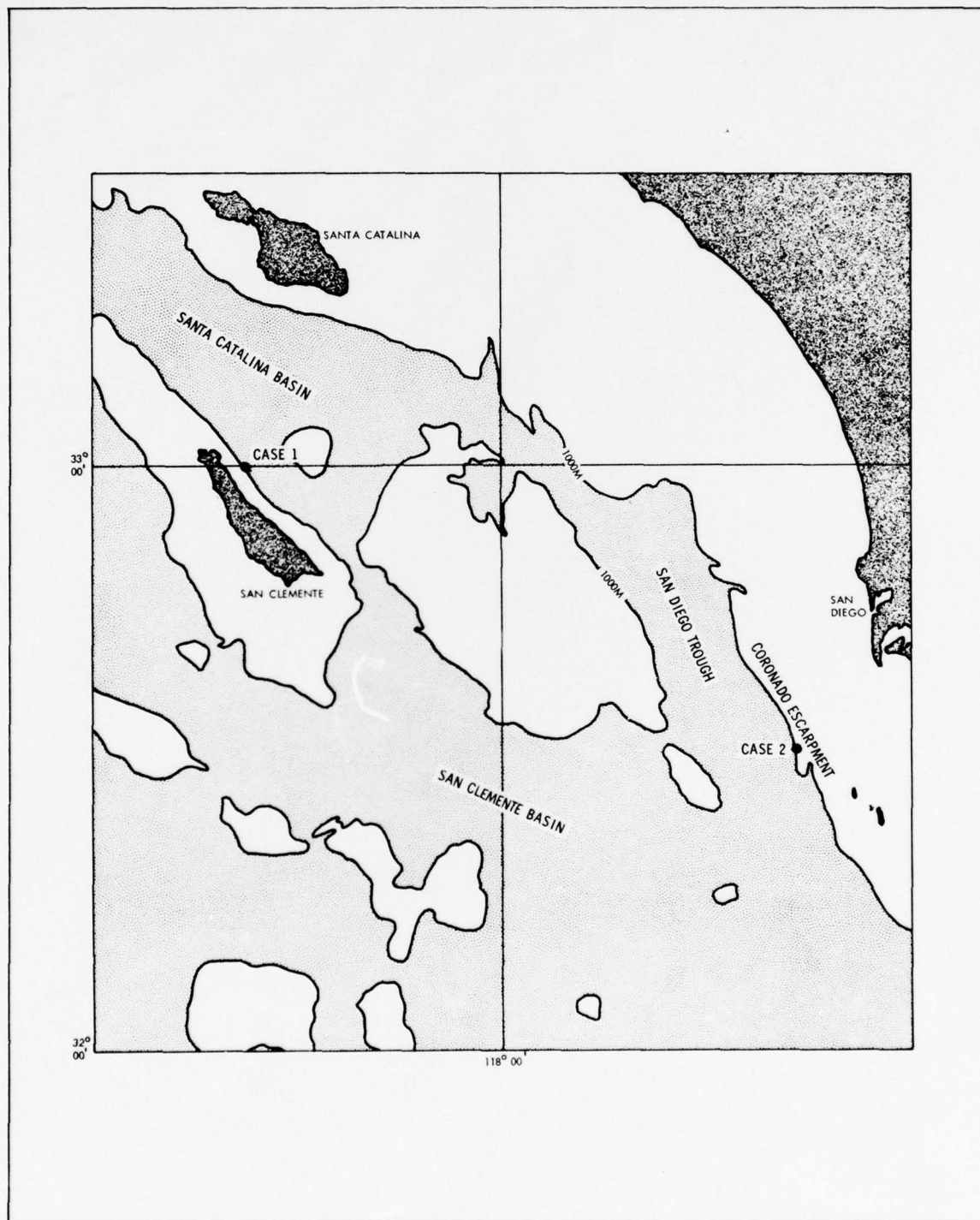


Figure 1 - Sites and Exposures of the Observations. The dotted area delineates depths greater than 1000 m.

and

$$[S(L)]^2 = \frac{1}{N(L)} \sum_{i=1}^{N(L)} [T(i) - T(i+L)]^2$$

where

L is the temporal lag factor

$N(L)$ is the total number of useful pairs of observations for a given L

i is the serial number of the observation

$T(i)$ is the i th temperature observation

The well known statistical properties of the standard deviation of a normal distribution cause $S(L)$ to be useful in studying the distribution of the temperature differences, since $S(L)$ is simply the standard deviation of the temperature difference at lag L . Some of the results of these computations are shown in Figures 2 and 3.

These figures are a graphical display of the great thermal unrest in the ocean. These observations were not taken in a geographical region of known exceptionally large variability. Rather, they are thought to be fairly typical situations. At lag values under about 10 min the root-mean-square successive differences, $S(L)$, are easily represented by a simple relationship of the type $S(L) \propto L^b$. This tends to confirm the results of Skudrzyk [1963] and others in which the small-scale temperature variations were shown to follow the Kolmogorov equilibrium range similarity hypothesis of turbulence.

The autocorrelation function was computed according to the expression:

$$R(L) = \frac{N(L) \sum T(i)T(i+L) - \sum T(i) \sum T(i+L)}{\left\{ N(L) \sum T(i)^2 - [\sum T(i)]^2 \right\}^{1/2} \left\{ N(L) \sum T(i+L)^2 - [\sum T(i+L)]^2 \right\}^{1/2}}$$

where in all summations $i = 1, 2, \dots, N(L)$

Figures 4 and 5 present these results for Cases 1 and 2, respectively. If $S(L)$ had been known for large L , then a separate computation would not have been necessary.

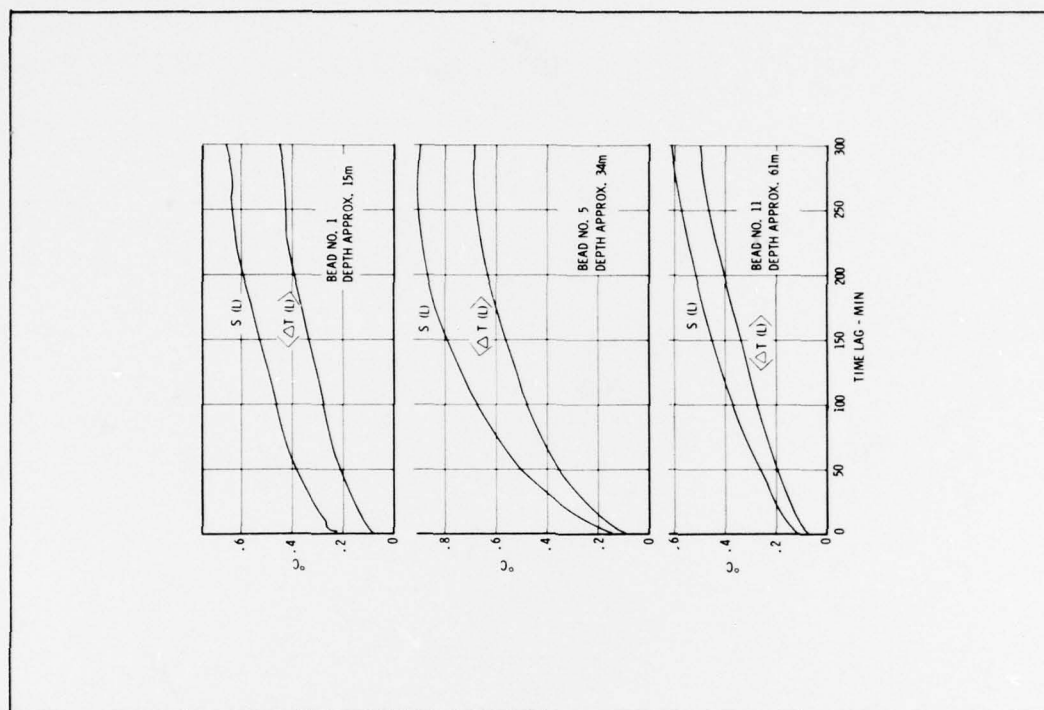


Figure 3 - Typical Temperature Change as a Function of Time Lag Between Observations. Case 2. See Fig. 2 for explanation.

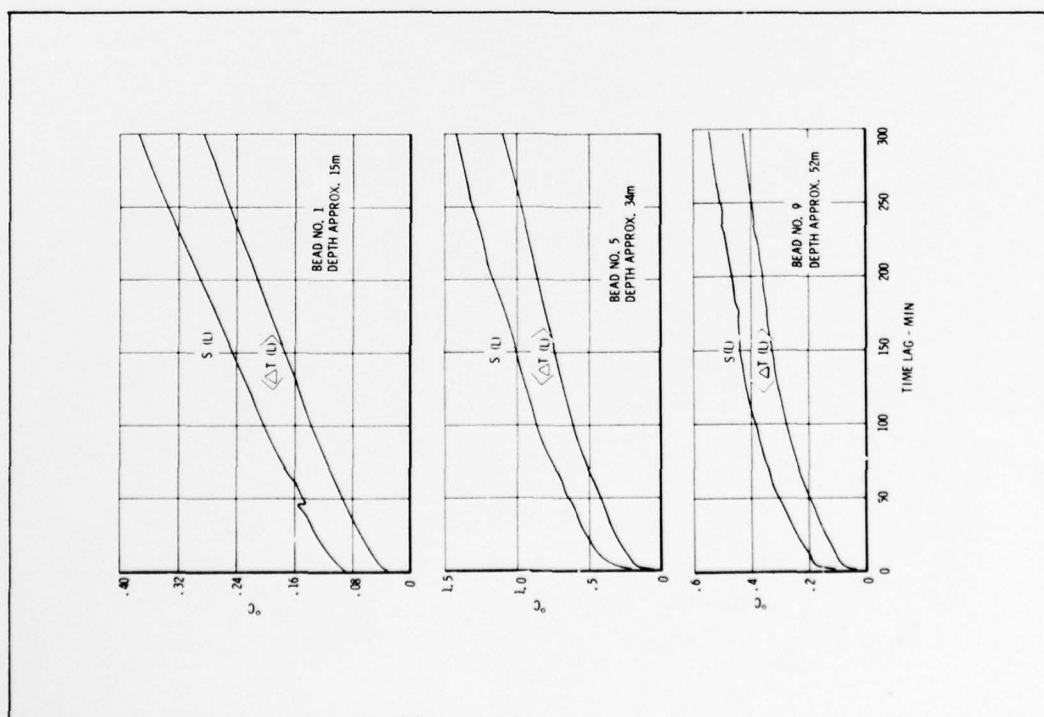


Figure 2 - Typical Temperature Change as a Function of Time Lag Between Observations. Case 1. $S(L)$ is the root-mean-square successive deviation, $\langle \Delta T(L) \rangle$ is the mean absolute successive difference.

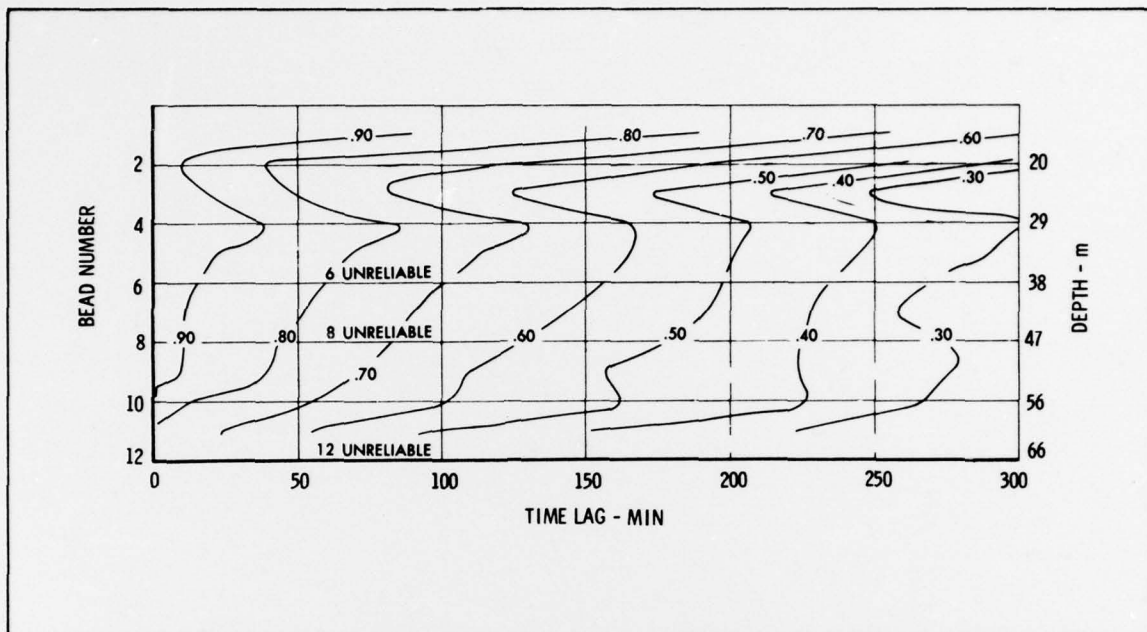


Figure 4 - Autocorrelation of the Temperature-Time Series. Case 1

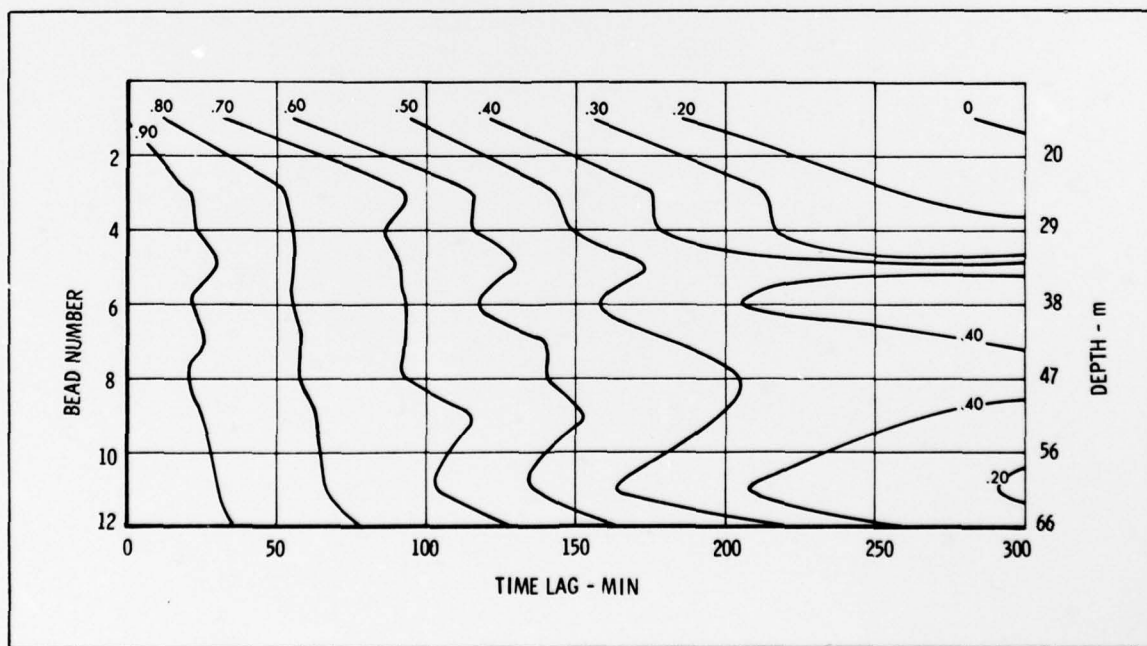


Figure 5 - Autocorrelation of the Temperature-Time Series. Case 2

The autocorrelation of Figure 4 is seen to decrease generally with depth, indicating that more random processes are causing the temperature variations in the deeper layers. Figure 2 shows that the temperature variation recorded by bead 9 is much less than the variation of bead 5 where a reasonably high correlation exists. Thus, the decrease in correlation could be due to an adverse signal-to-noise ratio. This decrease is consistent with the results presented by LaFond and Moore [1960, 1961], which used bathythermographs in which the error in depth is roughly proportional to the depth. In this case, the noise caused by errors increases while the variance of the real changes decreases. Both tend to decrease the signal-to-noise ratio.

The other significant feature shown in Figure 4 is the relatively high correlation near the top of the thermocline. Figure 5, which presents the autocorrelation for Case 2, has a very different appearance. The figure shows the lower correlations at the top bead and the higher correlations at longer periods at 46 m.

There is a marked difference between Figures 4 and 5, even though the same instrumentation and techniques were used in both experiments. One explanation is that the thermocline was much sharper in Case 1 than in Case 2, thus, the apparent stability of the surface layer in Case 1 was much lower than in Case 2. This is shown by the temperature profiles plotted in Figures 6 and 7. These profiles are made up of the average value of the temperature over various representative 1-hr increments. The numbers written beside the hourly mean temperature curve are the respective standard deviations found over the hour. They are presented in this fashion because the existence of correlation precludes computation of a simple reliable confidence interval.

DISTRIBUTION

Knowledge of the frequency distribution of the temperature fluctuation is essential if we are to answer the question posed initially in this study. This distribution was found for lags of 10, 50, 90, 150, 225, and 300 min of both cases. Some of these results are presented in Figures 8 and 9. This was found simply from the raw data by computing

$$\Delta T(I, L) = T(I) - T(I+L)$$

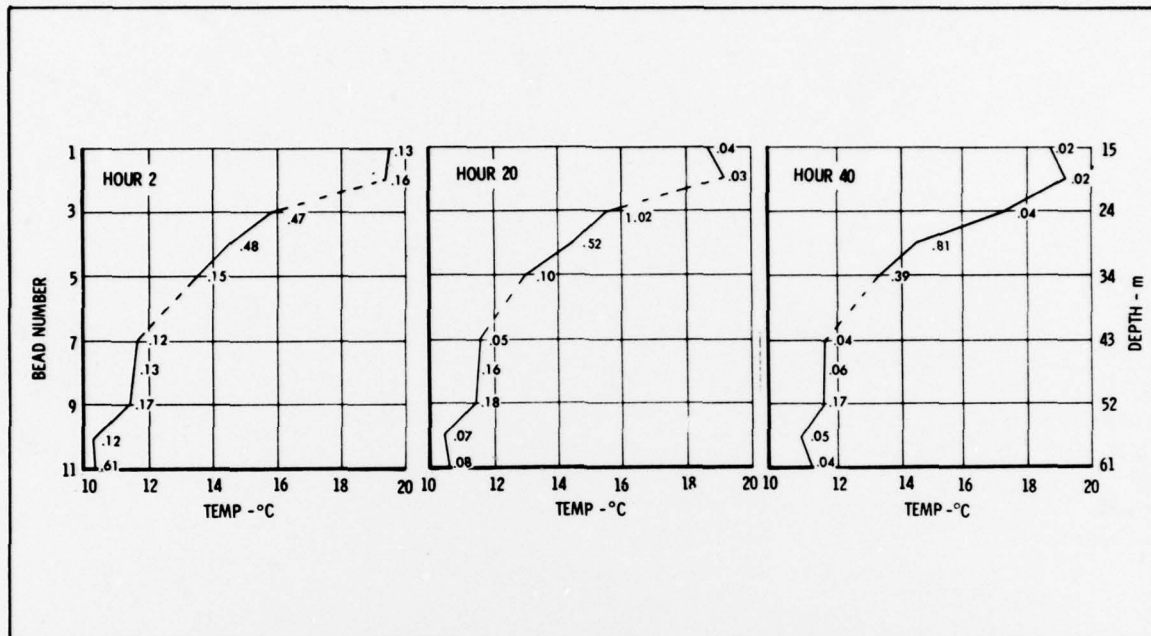


Figure 6 - Typical Hourly Mean Temperature Profiles. Case 1. The small numbers show the standard deviation of the observations over the hour. Dashed lines are used to connect regions of missing data.

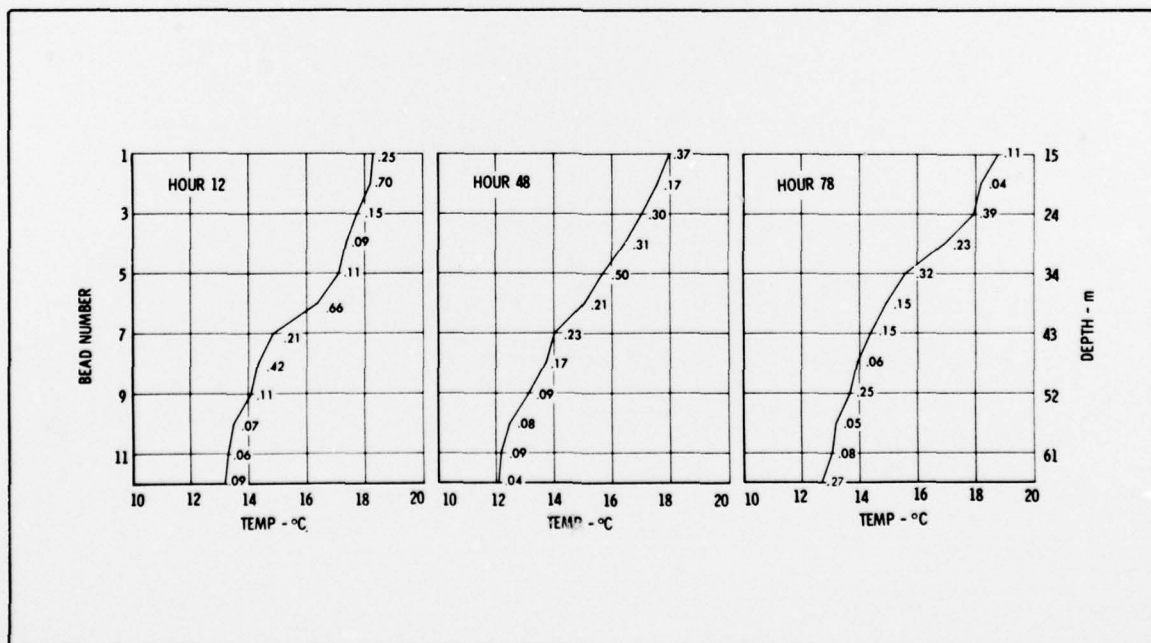


Figure 7 - Typical Hourly Mean Temperature Profiles. Case 2. See Fig. 6 for explanation.

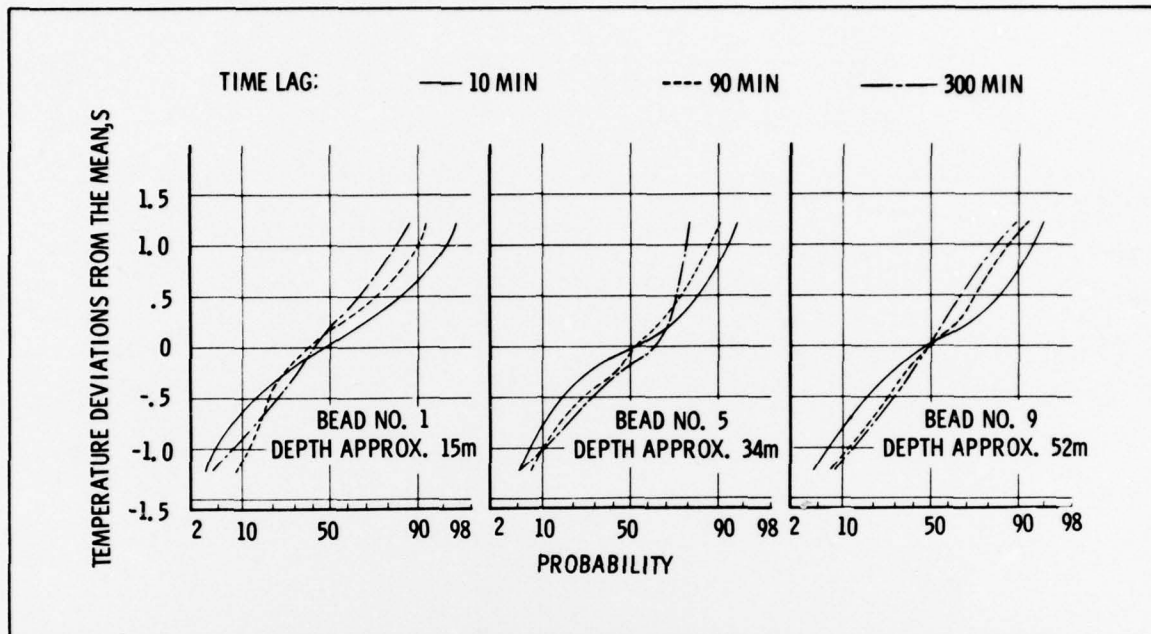


Figure 8 - Typical Frequency Distributions of Temperature Changes for Time Lags of 10, 90, and 300 Minutes. Case 1.

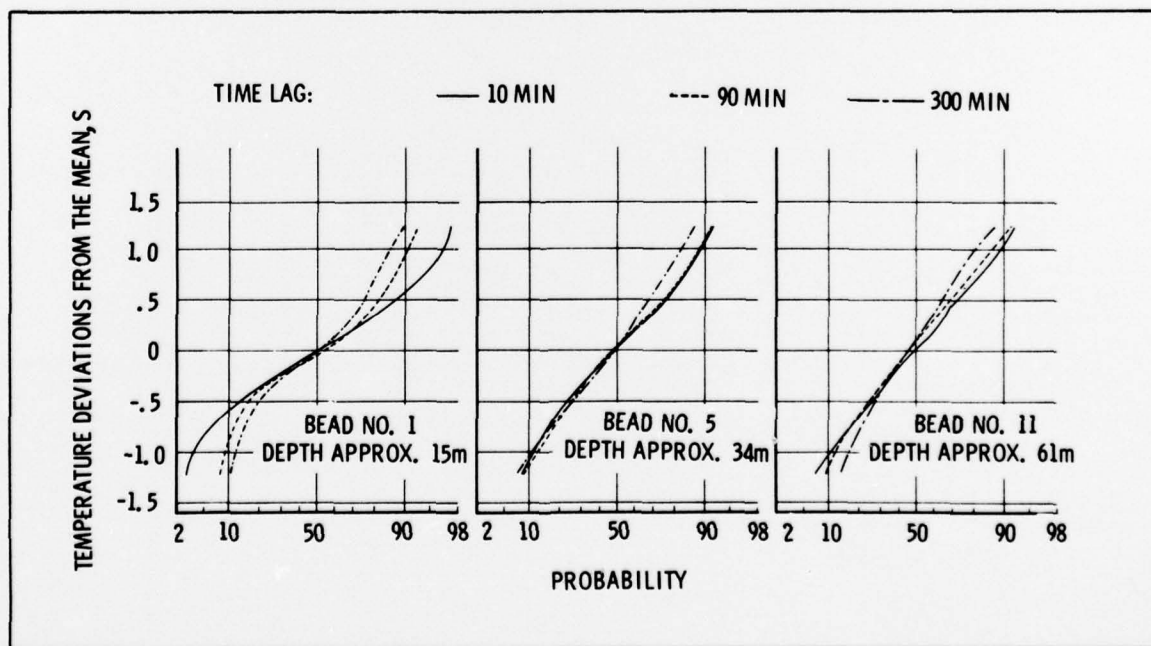


Figure 9 - Typical Frequency Distribution of Temperature Changes for Time Lags of 10, 90, and 300 Minutes. Case 2.

for $L = 10, 50, 100, 150, 225,$ and 300

$$I = 1, \dots, N(L)$$

and grouping the ΔT into nine intervals covering the range $-\infty$ to $+\infty$ for each K . The intervals were chosen so that the probability of occurrence would be a constant $1/9$ for each interval if the population were normally distributed. The mean values for $\Delta T(I, L)$ were close to zero. The computed distributions are plotted on normal distribution graph paper where a straight line represents a normal distribution.

As can be seen from Figures 8 and 9, all distributions are nearly symmetrical and most have high Kurtosis. That is to say that the probability of occurrence of the larger temperature differences is seen to be much lower than that expected of a normal distribution. This is especially pronounced in the shallower beads. The usual explanation for this is that the population is not homogeneous. Thus this distribution shown might be the result of adding two or more distributions having significantly different variances. This hypothesizes that two independent regimes either alternate in occurrence or exist simultaneously. The fact that this effect is more pronounced near the surface could mean that some higher internal wave modes are occurring. Other hypotheses might be that diurnal temperature changes, surface waves, or other external processes are important in causing the perturbations which may be either waves, inhomogeneities, or cellular motions.

COHERENCE

To learn the interdependence of the temperatures at different depths we computed the coherence for bead record pairs (1, 5) and (5, 9) for Case 1 and (1, 5) and (5, 11) for Case 2. The method used was essentially that described by Goodman [1957]. The squared coherence function $C_o^2(\omega)$ of the temperatures between depths a and b is given by the expression

$$C_o^2(\omega) = [C^2(\omega) + Q^2(\omega)]/S_a(\omega) S_b(\omega)$$

where $C(\omega)$ and $Q(\omega)$ are the cosine and the sine transformations (the co-spectrum and the quadrature spectrums respectively) of the cross-correlation between the temperatures

at depths a and b . $S_a(\omega)$ and $S_b(\omega)$ are the spectra of the temperatures at the respective depths a and b . The argument ω is the angular frequency of the above functions. In much the same manner as the square of the common correlation coefficient is a measure of the degree of linear dependence between two random variables, the coherence is a measure of the degree to which two random processes are linearly related. It is unity if they are in a perfect linear relation and zero if they are linearly independent.

For all frequencies of each case the coherence was found to be small. The simple interpretation then is that most of the perturbations were smaller in extent than about 20 m. However, because complicated motions of the chain are possible, such a simple interpretation may be suspect. The low coherence also indicates that the observation chain did not have a significant regular vertical oscillation greater than 2-min, the lower limit of observation. If the chain had moved consistently in a long-period vertical oscillation, the sensors would have recorded similar excursions and the value of the coherency for that frequency would have been significant.

ALIASING

Much of the previously published work has been directed toward the longer-period variations by using a time series observed at widely spaced intervals. Thus, the aliasing of high-frequency components into the lower frequencies could not be avoided. To demonstrate this phenomena and show a method of alleviating the misrepresentation it causes, we prepared a new time series of hourly average values. This process had the additional advantage of reducing the effect of instrumental errors.

Both aliasing from high frequencies and observational errors tend to increase the noise level of the signal at the longer periods. It follows then, that an averaged or filtered series is a method of providing more meaningful estimates of the longer periods. Ideally, a different, carefully shaped filter should be used for each frequency considered. However, for purposes of illustration, a simple averaging procedure was used.

Figure 10 shows the autocorrelation of the averaged time series derived from Case 2. A definite fluctuation having about a 12-hr period is easily seen. This corresponds roughly to both the semi-diurnal tidal period and the inertial period at the latitude of observation. Generally, for lags less than 5 hours, where they can be compared, the autocorrelations of the averaged series are, as expected, higher than those

of the unfiltered series shown in Figure 5. Figure 11 compares the root-mean-square deviations. Again, the averaged or filtered series shows smaller deviations than the unfiltered, and maxima occur at 6 hr intervals corresponding to fluctuations of 12-hr periods. If there were only a 12-hr period present, the autocorrelation would be near zero at 6 and 18 hr, and the root-mean-square deviation zero at 3 and 9 hr. At both the shallowest and deepest depths this is approximately the case; but, at the intermediate depths, there is at least one other wavelength present.

The periods of less than 2 hr were crudely filtered from the original series by the hourly averages. For Case 2, bead 5, only about 25% of the amplitude of the root-mean-square successive differences was lost in the process. Thus, there are significant long-period fluctuations which are perhaps more easily seen by the averaging process. A second averaging of the record was performed this time over 6-hr intervals. The result is also shown in Figure 11. This decreased the magnitude of the hour averages by about 40% at lags of 12 hr or less. Thus, slightly more than half the magnitude of the fluctuations is due to periods of less than 12 hr.

To summarize, aliasing will cause biased results. It can be avoided by not using instantaneous temperatures. Only if the small-scale variability is known from observations, a theoretical or empirical model, or some other means, can the bias be subtracted from the larger-spaced record and a realistic estimate of the errors be made. Knowledge of these small-scale phenomena would allow an optimum filter to be designed which would be more efficient than the averaging device used [Peterson and Middleton, 1963].

RELIABILITY OF ESTIMATES

In the previous sections of this paper, we have ignored sampling variability and measurement errors. However, they must be considered. The resultant accuracy of the individual temperature measurements was given by Carsola, et al, [1963] to be $\pm 0.1^\circ\text{C}$ in the range of 13° to 14° . If the measurement errors follow a normal distribution with a zero mean and the $\pm 0.1^\circ\text{C}$ error is interpreted to be the "3 σ " level for all temperatures, we can compute the errors in each successive difference to have a variance, $S_e^2 \leq 0.002$, because of positive correlation.

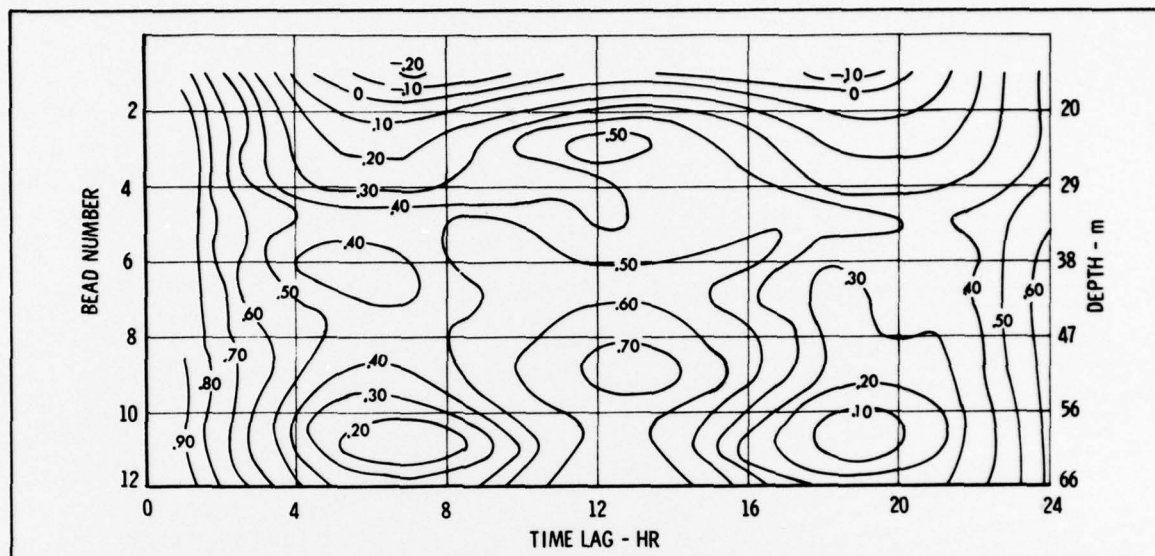


Figure 10 - Autocorrelation of the Filtered Temperature-Time Series. Case 2.

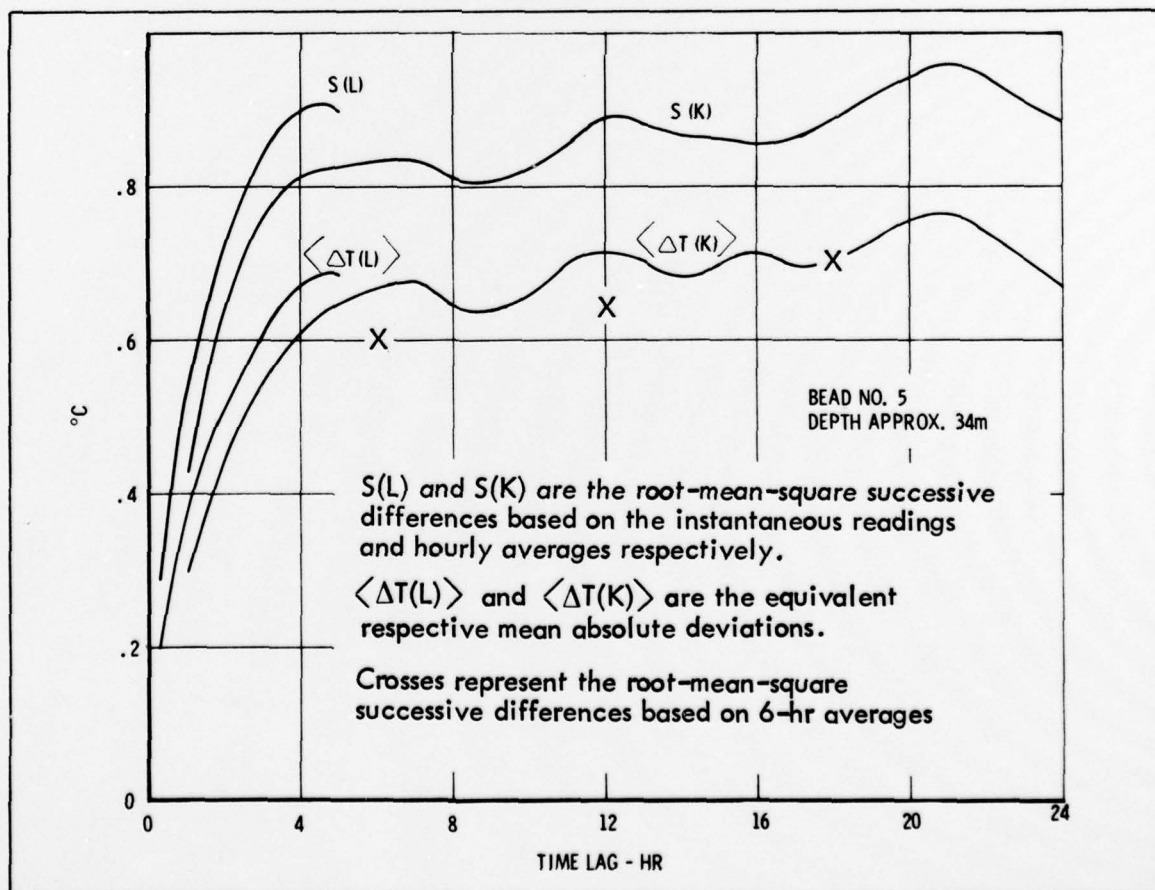


Figure 11 - Comparison of Filtered and Unfiltered Root-Mean-Square Successive Deviations. Case 2.

Since these errors and the observations are independent,

$$S^2 = S_e^2 + S_a^2$$

where S_a is the true standard deviation and S is the value based on observations presented previously. Thus, the square of the values plotted is too large by not more than 0.002, and the true values, S_a , are only slightly smaller than the S presented. It can also be shown that the sampling variability described by $\text{var}[S_e^2]$ is also small.

Confidence intervals on the autocorrelation function, $R(L)$, can be roughly approximated by using the standard Fisher's Z transformation when the number of independent observations N' is taken as $N/2$ to allow for lack of independence breaks in the record, and the fact that at the longer lags fewer pairs are available. We let

$$Z = 0.5 \ln \left[\frac{1 + R(L)}{1 - R(L)} \right]$$

Then $\text{var}[Z] = \frac{1}{N' - 3}$

$$\hat{\sigma}_Z = \sqrt{\text{var } Z} = \begin{cases} 0.0247 \text{ for Case 1} \\ 0.0190 \text{ for Case 2} \end{cases}$$

or $\hat{\sigma}_Z \approx 0.02$

The corresponding 95% confidence intervals for R are shown in Table I.

Estimation of confidence intervals of the computed root-mean-square deviations, $S(L)$, is somewhat complicated. However, a rough approximation can be made simply. By algebraic manipulation on the definition of $S(L)$, we obtain

$$\frac{S^2(L)}{\sigma^2} = 2[1 - R(L)]$$

where $\sigma^2 = \text{var.}[T]$. The distribution of $S^2(L)/\sigma^2$ is thus simply related to the distribution of $R(L)$. Assuming the variation of $\hat{\sigma}$ itself to be small, the confidence intervals on $S(L)$ can be found. To take an extreme example with a correlation of 0.9 and an $S(L) = 0.3$, $\sigma^2 = .45$ and the 95% confidence interval is within about $0.3 \pm 0.015^\circ\text{C}$. This is quite small as one would expect because of the large sample sizes from which the S 's were computed.

It is a proper question to ask if the 1-min sampling interval is adequate or whether significant aliasing was caused by its use. There is no simple, absolute answer to such a question. However the fact that only small variations occur over the shortest lags, as shown in Figures 2 and 3, lends some support to its adequacy. Though most of the values are small, some, especially those from observations taken nearest the surface, are still large enough to provide some doubts. For future research an investigation using even smaller observation intervals would be worthwhile.

TABLE I
RELIABILITY OF R

COMPUTED R(L)	95% RANGE OF R(L)	
	UPPER	LOWER
.3	.33	.26
.5	.53	.47
.7	.72	.68
.9	.91	.89

SECTION III

CONCLUSIONS

Results of this study provide a partial answer to the question, "how representative is an oceanic temperature observation?" This answer must vary with geographic location and season, therefore these results applies only to the specific location, and time of the observations reported herein. To arrive at the solution, we computed the root-mean-square consecutive differences, $S(L)$, and the frequency distribution in terms of $S(L)$ for various lags, L . From these two, the probability of temperature variations exceeding any given level can be computed. As an example, Figures 12 and 13 have been prepared to show the recorded temperature deviations which were exceeded 20% of the time for various lags and depths for Cases 1 and 2. This type of graph can describe the temperature variability; be used to design observing systems; or to test the real accuracy of temperature predictions. Specifically, Figure 13 shows, for example, that there is a 20% chance that two perfect, instantaneous observations taken 3 hr apart at about 30-m depth will disagree by at least 1°C . Thus, in this case, 20% of the measurements will not describe the temperature 3 hr later with a greater accuracy than 1°C no matter how good the sensor.

The temporal and spatial sampling interval must be consistent with the sensor accuracy for an efficient system. Thus, we must ask if it is worthwhile to use near-instantaneous sensing and whether the high accuracy of an instantaneous sensor is worth the extra cost.

Because the temperature changes which take place over the first few minutes are significant when compared to the changes which take place over several hours, consideration of the small-scale effects must be made to eliminate aliasing so that the larger synoptically significant variations are not obscured. It is almost impossible to forecast the small-scale fluctuations. However, a low-pass filtered series might be predictable. In other words, it is more feasible to predict the hourly average than the hourly value of temperature. Therefore, it might be worthwhile to observe the averages directly.

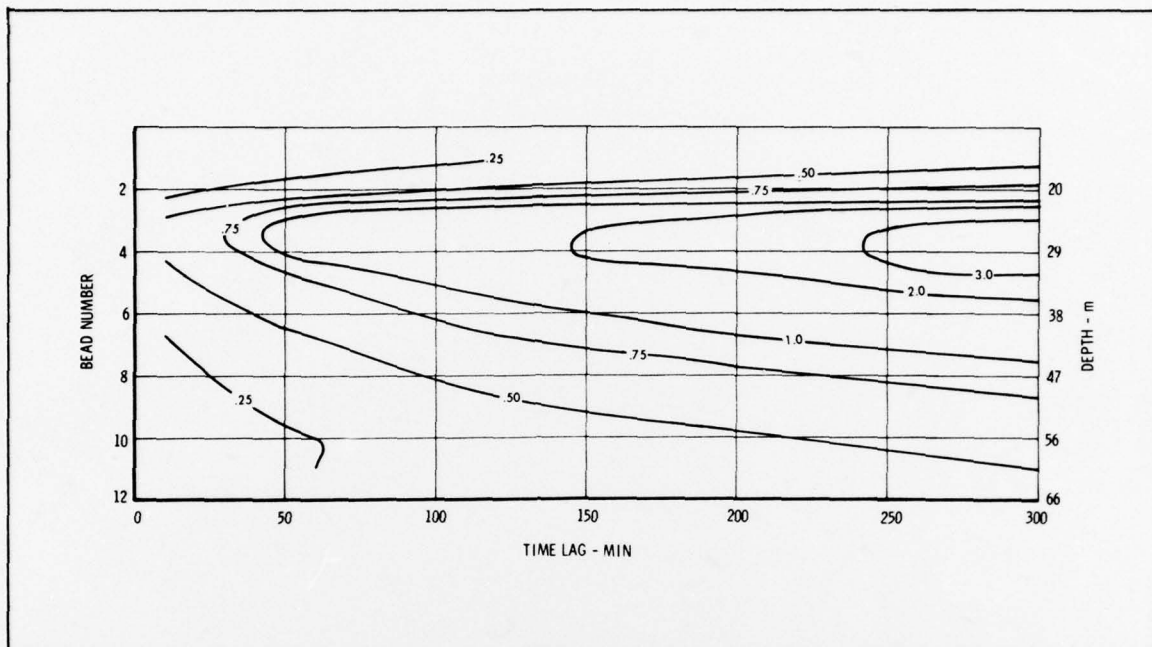


Figure 12 - Temperature Change as a Function of Time Lag Between Observations That Was Not Exceeded in 80% of the Observations. Case 1.

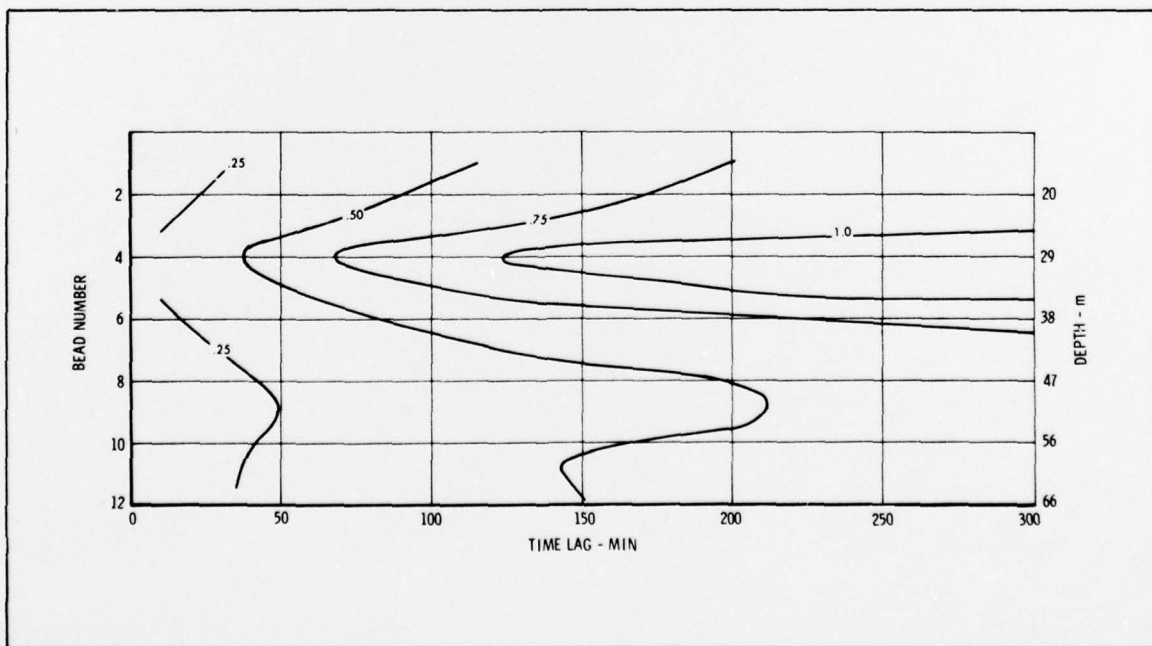


Figure 13 - Temperature Change as a Function of Time Lag Between Observations That Was Not Exceeded in 80% of the Observations. Case 2.

There are several methods of carrying this out. The simplest is to increase the time lag of the sensor by enclosing it in a bag or other covering. This has the disadvantage of yielding a non-linear average as do the simpler electronic integrators. However, relatively simple and inexpensive mechanical integrators which can be built from a standard counter could be used. In this case the filter is shaped by specifying the times that the sensors are read. A pair of integrators with overlapping times would be most efficient since observations must be taken at intervals of one-half period while filters require a full period.

Since the number recorded is the sum of many individual readings, the accuracy of the sensor need not be great. It is important that each reading of the sensor be unbiased and without drift. Thus, a reasonable system to observe the hourly and longer fluctuations might be one based on a relatively inexpensive thermistor or other sensor that had an error of one standard deviation equal to 0.03°C . The average of 30 samples which are taken at specified times from 30 minutes before the observation time to 30 minutes after would then be recorded. The sensor might have a time constant of a minute. The sample times chosen would be biased to give more readings near the center of the interval than near the extremes. By the central limit theorem of classic statistics, the error in the average will have a standard deviation of less than 0.0055°C which is adequate for synoptic purposes.

The knowledge of temperature variabilities such as have been investigated in this paper is essential for efficient operation of underwater acoustic systems, synoptic networks, and similar apparatus. Without this knowledge one would expect accuracies which could never be attained with any reasonable consistency because of unknown temperature perturbations. Whenever ocean temperature is to be used as a parameter, both the small-scale temporal and spatial variability must be investigated to understand the larger-scale variations.

The primary purpose of this paper is to arouse interest in this basic problem of representativeness of observations. Further study would require several three-dimensional arrays of sensors, so that both spatial and temporal changes could be studied simultaneously. The work should then be extended to the study of other depths, oceanic areas and seasons. The variability of other parameters such as salinity, and current velocity must also be investigated.

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